Trans. Circuit Theory, vol. CT-5, pp. 104–109, June 1958.
[4] W. J. Getsinger, "Coupled rectangular bars between parallel plates," *IRE Trans. Microwave Theory Tech.*, vol. MTT-10, pp. 65–72, Jan. 1962.
[5] J. F. Reynolds *et al.*, "Coupled TEM bar circuit for *L*-band silicon avalanche oscillators," *IEEE J. Solid-State Circuits*, vol. SC-5, pp. 346–353, Dec. 1970.
[6] A. S. Clorfeine *et al.*, "High-power widebandwidth TRAPATT circuits," *IEEE J. Solid-State Circuits*, vol. SC-10, pp. 27–31, Feb. 1975.
[7] T. T. Fong *et al.*, "Fixed-tuned high power *F*-band TRAPATT amplifier," in 1977 *IEEE Int. Solid-State Circuits Conf. Dig. Tech. Pap.*, pp. 124–125 Feb. 1977.
[8] ——, "*F*-band TRAPATT amplifier," for Naval Electron. Systems Command, Washington, D.C., Contract no. N0039-75-C-0081 Final Rep.

Techniques for Broad-Banding Above Resonance Circulator Junctions Without the Use of External Matching Networks

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Abstract—By using a modification of Bosma's approach, a theoretical explanation is given for the broad-banding effect which can be achieved by the use of three open-circuited stubs on the center conductor circumference of above resonance circulators. It is also shown that similar broad-banding can be achieved by the use of materials with very large values of $4\pi M_s$ in the above resonance state. It appears that the frequency dependence of the conductance $G(\omega)$ is a limiting factor in the bandwidth improvements that can be obtained in these ways.

I. Introduction

Above resonance circulators are normally used at low microwave frequencies where, for reasons of compactness, it is desirable to obtain as large an intrinsic bandwidth as possible without the use of external matching networks. One common broad-banding technique, which has however until now not received a theoretical explanation in the literature, is to use three open-circuited stubs around the center conductor circumference. In this contribution a theoretical explanation based on Bosma's approach is given. Furthermore, it is shown experimentally that similar broad-banding can be achieved by using the standard junction geometry but materials with very large values of $4\pi M_s$. Both approaches reduce the susceptance slope parameter of the junction susceptance while leaving the frequency dependence of the conductance $G(\omega)$ nearly unaffected. It is found that this frequency dependence is substantial for above resonance circulators and that $G(\omega) \propto \omega^{\nu}$ where $\nu \geqslant 2 (kR \cong 1.84)$.

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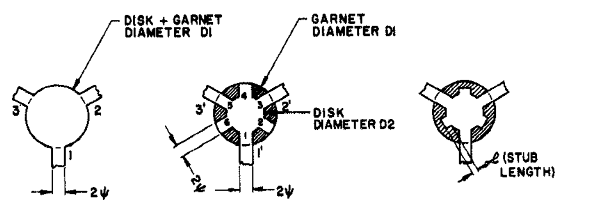

Fig. 1. Various stripline circulator geometries considered in the text.

II. Broad-Banding with Stubs

The diagram of a conventional circulator junction is given in Fig. 1(a). The center conductor diameter and garnet (ferrite) diameter are essentially the same while each of the striplines subtends an angle of 2ψ with the garnet circumference. A version of the geometry we will be considering is given in Fig. 1(b). The center conductor diameter is reduced while the garnet diameter remains the same. Three stubs situated between the connecting striplines and extending out to the garnet boundary have been added. Although it is possible to perform calculations if the stubs have a different width than the striplines, it will be assumed here that their widths are the same. A further modification that is readily treated is given in Fig. 1(c) where the stubs do not extend to the garnet boundary. As a typical example for the purpose of illustration we will assume a garnet disk 1 in in diameter with $\epsilon = 15.0$ and $4\pi M_s = 0.10$T. Furthermore, it will be assumed that $\phi = 0.25$ rad at the garnet boundary corresponding to a ground plane spacing of about 0.150 in.

The sort of problem we are considering can be treated using a modification of Bosma's approach by considering the basic junction to be a six-port device. The boundary conditions at the center conductor disk circumference analogous to those of Bosma [1] are

$$\begin{aligned} -\theta<\phi<\theta, &\quad H_\phi=H_1\\ 60^\circ-\theta'<\phi<60^\circ+\theta', &\quad H_\phi=H_2\\ 120^\circ-\theta<\phi<120^\circ+\theta, &\quad H_\phi=H_3\\ 180^\circ-\theta'<\phi<180^\circ+\theta', &\quad H_\phi=H_4\\ 240^\circ-\theta<\phi<240^\circ+\theta, &\quad H_\phi=H_5\\ 300^\circ-\theta'<\phi<300^\circ+\theta', &\quad H_\phi=H_6\\ &\quad H_\phi=0 \text{ elsewhere} \end{aligned} \tag{1}$$

where 2θ (the angular width of the connecting strips at the center conductor circumference) and $2\theta'$ (the angular width of the stubs at the center conductor circumference) are greater than 2ψ. With these boundary conditions the impedance matrix of the six port can be found. The three eigenadmittances of the three port formed by terminating ports 2, 4, and 6 in open-circuited stubs of known length can then be determined. Then these eigenadmittances must be transformed through the transforming section extending from the center conductor disk to the garnet disk boundary. From these eigenadmittances the equivalent admittance at the garnet boundary can be calculated using known formulas [2]. This establishes the basic approach.

In practice there are many free parameters: The garnet diameter, the widths of the striplines and stubs, the lengths of the stubs and transforming sections. In what follows the simplifying assumption will be made that the striplines and stubs have the same width. This assumption reduces the number of free parameters and simplifies the mathematics while still allowing the broad-banding effect to be clearly demonstrated. In fact with this assumption our basic network becomes a fully symmetrical six port for which expressions for the eigenimpedances given by Davies and Cohen can be used [3].

If S is the S matrix of the symmetrical six port, then $\vec{V}'=S\cdot\vec{V}$ where $\vec{V}$ is the incident voltage vector and $\vec{V}'$ is the reflected voltage vector. Taking S_{11}, S_{12}, S_{13}, S_{14}, S_{15}, and S_{16} as the independent S matrix entries, then for the eigenexcitations

$$\begin{pmatrix} V_1'(0)\\ V_4'(0)\end{pmatrix}=\begin{pmatrix}\{S_{11}+S_{13}+S_{15}\} & \{S_{12}+S_{14}+S_{16}\}\\ \{S_{12}+S_{14}+S_{16}\} & \{S_{11}+S_{13}+S_{15}\}\end{pmatrix}\begin{pmatrix}V_1(0)\\ V_4(0)\end{pmatrix} \tag{2}$$

$$\begin{pmatrix} V_1'(1)\\ V_4'(1)\end{pmatrix}=\begin{pmatrix}\{S_{11}+S_{13}e^{j2\pi/3}+S_{15}e^{-j2\pi/3}\} & \{S_{12}e^{-j2\pi/3}+S_{14}+S_{16}e^{j2\pi/3}\}\\ \{S_{12}e^{-j2\pi/3}+S_{14}+S_{16}e^{j2\pi/3}\} & \{S_{11}+S_{13}e^{j2\pi/3}+S_{15}e^{-j2\pi/3}\}\end{pmatrix}\begin{pmatrix}V_1(1)\\ V_4(1)\end{pmatrix} \tag{3}$$

$$\begin{pmatrix} V_1'(-1)\\ V_4'(-1)\end{pmatrix}=\begin{pmatrix}\{S_{11}+S_{13}e^{-j2\pi/3}+S_{15}e^{j2\pi/3}\} & \{S_{12}e^{j2\pi/3}+S_{14}+S_{16}e^{-j2\pi/3}\}\\ \{S_{12}e^{j2\pi/3}+S_{14}+S_{16}e^{-j2\pi/3}\} & \{S_{11}+S_{13}e^{-j2\pi/3}+S_{15}e^{j2\pi/3}\}\end{pmatrix}\begin{pmatrix}V_1(-1)\\ V_4(-1)\end{pmatrix} \tag{4}$$

where 0, -1, and 1 refer to the three eigenexcitations and it is assumed that ports 2, 4, and 6 are terminated in the same reactive termination to maintain three-fold symmetry. For each eigenexcitation there is an equivalent symmetrical two port S matrix of the form

$$S(i)=\begin{pmatrix} S_{11}(i) & S_{12}(i)\\ S_{12}(i) & S_{11}(i)\end{pmatrix},\qquad i=0,-1,1. \tag{5}$$

Clearly if the transfer matrices corresponding to these two port S matrices can be found, then the eigenadmittance for each eigenexcitation can be found by multiplying together the transfer matrix for the stub, the transfer matrix for the equivalent two port, and the transfer matrix for the transforming section (see Fig. 2) and by assuming the resulting network to be terminated in an open circuit.

We would like to obtain the elements of the equivalent transfer matrices for the three eigenexcitations in terms of the eigenimpedances of the six port for which expressions exist in the literature. If we let $S_0(i)=S_{11}(i)+S_{12}(i)$, $S_1(i)=S_{11}(i)-S_{12}(i)$ $(i=0,-1,1)$, then they are eigenvalues of the three symmetrical S matrices given by (5). It is then easy to show using (2)–(4) that $S_0(0)=S_0$, $S_1(0)=S_3$, $S_0(1)=S_{-2}$, $S_1(1)=S_1$, $S_0(-1)=S_2$, and $S_1(-1)=S_{-1}$ where the S_i's $(i=0,\pm1,\pm2,3)$ are the S matrix eigenvalues of the symmetrical six port. The eigenimpedances must satisfy the same relations so that

$$\begin{aligned} Z_{11}(0)&=\frac{Z_0(0)+Z_1(0)}{2}=\frac{Z_0+Z_3}{2}\\ Z_{12}(0)&=\frac{Z_0(0)-Z_1(0)}{2}=\frac{Z_0-Z_3}{2}\\ Z_{11}(1)&=\frac{Z_0(1)+Z_1(1)}{2}=\frac{Z_{-2}+Z_1}{2}\\ Z_{12}(1)&=\frac{Z_0(1)-Z_1(1)}{2}=\frac{Z_{-2}-Z_1}{2}\\ Z_{11}(-1)&=\frac{Z_0(-1)+Z_1(-1)}{2}=\frac{Z_2+Z_{-1}}{2}\\ Z_{12}(-1)&=\frac{Z_0(-1)-Z_1(-1)}{2}=\frac{Z_2-Z_{-1}}{2}. \end{aligned} \tag{6}$$

These equations can now be used to determine the transfer matrices for the three cases.

If the stub length is the same as the transformer length (as in Fig. 1(b)), then the situation simplifies still further

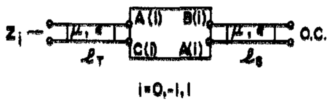

Fig. 2. The equivalent two-port network used to calculate the eigenimpedances Z_0, Z_{-1}, Z_1 for the geometries given in Figs. 1(b) and (c).

and simple expressions can be written down for the three eigenimpedances. The network given in Fig. 2 is now symmetrical so that with the output open-circuited,

$$Z_i = Z_{11}'(i) = \frac{Z_0'(i) + Z_1'(i)}{2}, \qquad i = 0, 1, -1$$

$$= \frac{j}{2}\left\{\frac{Z\tan\theta + Z_0(i)}{1-(\tan\theta/Z)Z_0(i)} + \frac{Z\tan\theta + Z_1(i)}{1-(\tan\theta/Z)Z_1(i)}\right\} \quad (7)$$

where θ is the electrical length of the stub or transforming section, $Z = \sqrt{\mu/\epsilon}$ and μ, ϵ are the magnetic and dielectric permeabilities of the garnet or ferrite material. Specifically,

$$Z_0(3) = \frac{j}{2}\left\{\frac{Z\tan\theta + Z_0(6)}{1-(\tan\theta/Z)Z_0(6)} + \frac{Z\tan\theta + Z_3(6)}{1-(\tan\theta/Z)Z_3(6)}\right\} \quad (8)$$

$$Z_1(3) = \frac{j}{2}\left\{\frac{Z\tan\theta + Z_{-2}(6)}{1-(\tan\theta/Z)Z_{-2}(6)} + \frac{Z\tan\theta + Z_1(6)}{1-(\tan\theta/Z)Z_1(6)}\right\} \quad (9)$$

$$Z_{-1}(3) = \frac{j}{2}\left\{\frac{Z\tan\theta + Z_2(6)}{1-(\tan\theta/Z)Z_2(6)} + \frac{Z\tan\theta + Z_{-1}(6)}{1-(\tan\theta/Z)Z_{-1}(6)}\right\}. \quad (10)$$

The number in parenthesis indicates whether the eigenimpedances refer to a symmetrical three-port or six-port device. The three-port eigenimpedances can now be used to determine the equivalent admittance [2]. The six-port eigenimpedances can be obtained with results summarized in the Appendix.

Calculations were performed using (8)–(10) assuming as a typical example a garnet disk ($\epsilon = 15.0, 4\pi M_s = 0.10$T) 1 in in diameter with $\psi = 0.25$ rad at the garnet circumference and center conductor diameters of a) 1.000 in, and b) 0.600 in. Cases a) and b) are given approximately to scale in Figs. 1(a) and (b), respectively. Graphs of theoretical conductance versus frequency and susceptance versus frequency for these cases are given in Figs. 3(a) and (b). It is apparent from Fig. 3(b) that introducing stubs significantly reduces the susceptance slope parameter. On the other hand the frequency dependence of the conductance is substantial and is only slightly effected. The significance of the frequency dependence of the conductance of above resonance circulators has been noted recently and will be discussed in more detail in a later section [4]. It is also apparent from these figures that introducing stubs lowers the center frequency and the required magnetic field for a garnet disk of constant diameter, which are both desirable features.

A further improvement could be achieved by reducing the length of the stubs with respect to the transforming

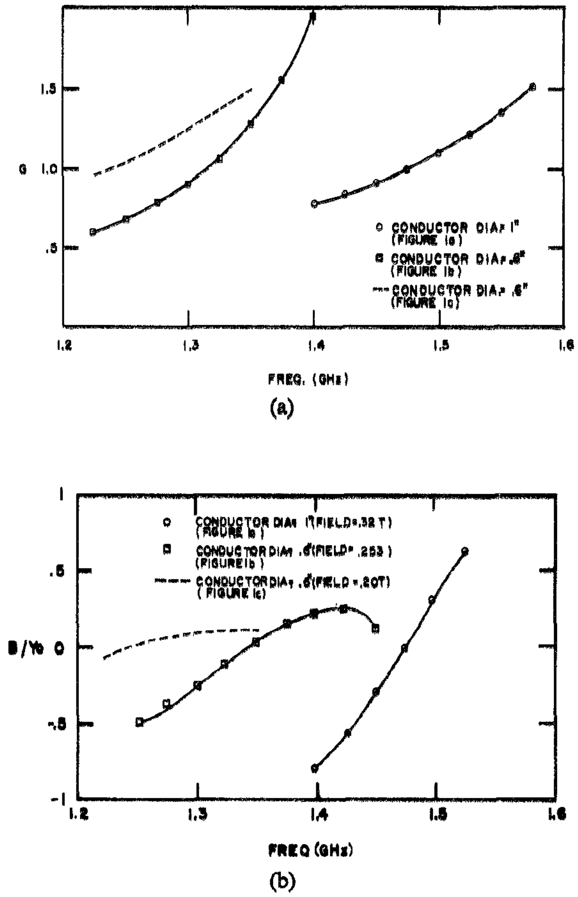

Fig. 3. The theoretical junction conductance and susceptance versus frequency for an above resonance circulator broad-banded with stubs (garnet diameter = 1 in, $4\pi M_s = 0.10$T, $\epsilon = 15$, and $\psi = 0.25$ rad) as in Figs. 1(b) and (c).

sections. The optimum situation seemed to be when the stubs are about half as long as the transforming sections (see Fig. 1(c)). The results for stubs 0.1 in long with a center conductor diameter of 0.6 in are given in Figs. 3(a) and (b) by the dashed lines. The susceptance is further flattened out as a function of frequency while the frequency dependence of the conductance is also somewhat reduced. Furthermore, the center frequency and required magnetic field (0.19T) are reduced still further.

III. Broad-Banding with Large Values of $4\pi M_s$

It is also possible to broad-band above resonance circulators while using the standard junction geometry of Fig. 1(a) by increasing the saturation magnetization $4\pi M_s$. As has been emphasized recently, the effect of increasing $4\pi M_s$ is to increase the effective magnetic permeability μ_{eff} at the center frequency [4]. The operating frequency range shifts to lower frequency but the absolute bandwidth in megahertz remains essentially constant. Consequently, for the same physical geometry the normalized bandwidth increases with increasing $4\pi M_s$. These points are demonstrated experimentally in Figs. 4 and 5 where return loss versus frequency is plotted for a circulator junction of the

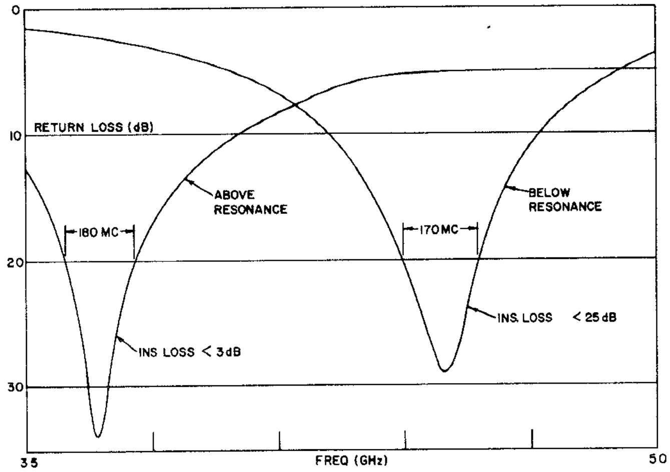

Fig. 4. Return loss versus frequency for a circulator junction using 1/2-in diameter X-506 disks ($4\pi M_s = 0.050$T, $p \cong 3$) biased above and below resonance. The center frequency is somewhat lower for the above resonance case because μ_{eff} is larger above resonance.

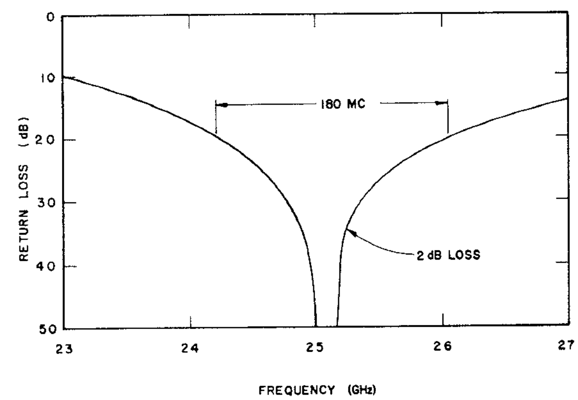

Fig. 5. Experimental return loss versus frequency for the junction of Fig. 4 but using 0.5-in diameter L3003 disks ($4\pi M_s = 0.30$T, $\epsilon \cong 15$) biased above resonance.

type of Fig. 1(a) ($\psi \cong 0.5$ rad) using in one case 0.5-in diameter X-506 disks ($4\pi M_s = 0.050$T, $\epsilon \cong 15$) and in the other 0.5 in diameter L3003 disks ($4\pi M_s = 0.30$T, $\epsilon \cong 15$). This last material is a lithium-ferrite material which is of some interest for above resonance circulators because of its good temperature properties ($T_c = 375°$C) and narrow line width ($\Delta H_{3dB} < 0.01$ T). Note that the absolute bandwidth is essentially constant for all cases. This broadbanding technique is useful when the coupling angle ψ is large so that the methods of Section II are inapplicable or where it is desirable to use a large ground plane spacing—as in high power applications. It has the disadvantage of requiring the application of a large magnetic field in order to bias the material above resonance.

IV. Comments on the Frequency Dependence of the Junction Conductance $G(\omega)$

Fig. 3(a) shows that the frequency dependence of the conductance for above resonance circulators is substantial in both the standard and broadband forms. It may become a limiting factor in the broadbanding which can be achieved. Recently, it was suggested that $G(\omega) \propto \omega^2$ above resonance [4]. These calculations assumed $\psi \cong 0.5$ rad whereas the dependence on ω is much stronger in Fig. 3(a) where $\psi = 0.25$ rad. In general in the frequency range where $kR \cong 1.84$ $G(\omega) \propto \omega^{\nu}$ where $\nu \geqslant 2$ and increases with decreasing coupling angle.

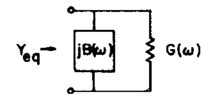

Fig. 6. The circuit form of the equivalent admittance for the junctions given in Table I.

The increase in the frequency dependence of G above resonance as the aperature angle decreases has implications for below resonance circulators as well. A major factor determining the frequency dependence of G is the frequency dependence of the tensor permeability component K. Well above resonance

$$K = \frac{4\pi M_s \gamma \omega}{\gamma^2 H_i^2 - \omega^2} \cong \frac{4\pi M_s \omega}{\gamma H_i^2}, \qquad \gamma H_i \gg \omega \tag{11}$$

whereas for an unsaturated material or well below resonance

$$K = -\frac{4\pi\gamma M}{\omega} \tag{12}$$

where $4\pi M$ = magnetization, $4\pi M_s$ = saturation magnetization, H_i = internal magnetic field, and $\gamma = 2.8$ MHz/Oe. Equations (11) and (12) together suggest that for the same physical structure the frequency dependence of the conductance will differ by a factor ω^2 between above and below resonance operation. Indeed if $\psi \cong 0.5$ and $G(\omega) \propto \omega^2$ above resonance, then G should be nearly frequency independent below resonance in agreement with the experimental facts. However, if the coupling angle ψ is small, then above resonance the frequency dependence of G will be much greater than ω^2 and will become significant below resonance as well. This is undesirable for broadband matching. Particularly for octave band circulators the coupling angle should not be too small. It should be noted that the wide band circulator of Wu and Rosenbaum uses a large coupling angle [5]. The proportionality factor is of course governed by the applied magnetic field. From (12) $|G|$ will increase with applied field below resonance until $M = M_s$ while from (11) $|G|$ will increase with *decreasing* field ($\gamma H_i > \omega$) above resonance.

A number of other reciprocal and nonreciprocal devices have an equivalent conductance (or resistance) which is a strong function of frequency. Typically the equivalent admittance is a frequency dependent conductance $G(\omega)$ shunted by a pure susceptance $jB(\omega)$ as in Fig. 6. In this case the frequency dependence of the conductance implies that the equivalent admittance is not a positive real function. Consequently, the attempt to broadband these devices with identical matching networks connected at each port constitutes a new kind of matching problem. Other devices falling into this category include lumped element

TABLE I
THE FREQUENCY DEPENDENCE OF $G(\omega)$ FOR VARIOUS MULTIPORT NETWORKS

Junction	Type	$G(\omega)$
3 Port Circulator	Stripline-below resonance ($kR\approx1.84$)	Constant
"	Stripline-above resonance ($kR\approx1.84$)	$\propto\omega^{\nu}(\nu>2)$*
"	Lumped Element-below resonance	$\propto 1/\omega^2$
"	Lumped Element - above resonance	Constant
Symmetrical 2 Branch Coupler	Uniform Transmission Lines ($\ell\approx\lambda/4$)	Constant
"	Shunt Connected Capacitors	$\propto\omega$
"	Shunt Connected Inductors	$\propto 1/\omega$
"	Series Connected Inductors	$R(\omega)\propto\omega$
"	Series Connected Capacitors	$R(\omega)\propto1/\omega$

* ν increases with decreasing coupling angle

circulators (above and below resonance) and symmetrical two branch couplers constructed using transmission lines or lumped elements [6]. For the lumped element circulator [7]

$$G(\omega)=\frac{\sqrt{3}}{\omega L}\frac{K}{\mu}. \tag{13}$$

It follows from (11) and (12) that $G(\omega)=$constant well above resonance and $G(\omega)\propto 1/\omega^2$ below resonance. It is perhaps fortunate that here, unlike the stripline circulator case, the above resonance mode of operation shows little frequency dependence since lumped element circulators are used at low microwave frequencies where above resonance operation is preferable. In the case of the symmetrical 2 branch lumped element coupler with the four ports connected by capacitors or inductors, $G(\omega)\propto\omega$ or $1/\omega$ [6]. This result applies for all frequencies. A summary of the frequency dependence of G for these different multiport networks is given in Table I.

V. Conclusions

Using a modification of Bosma's approach, a theoretical explanation has been given for a common technique for broad-banding above resonance circulators. It was found that this method reduces the susceptance slope parameter but has little effect on the frequency dependence of the conductance—which is substantial. For large coupling angles ($\psi\cong0.5$ rad) where this method can't be applied, it was demonstrated experimentally that broadbanding can still be obtained by using large values of $4\pi M_s$ for which certain lithium ferrites are of particular interest. Finally, it was emphasized that above resonance $G(\omega)\propto\omega^{\nu}$ where $\nu\geqslant2$ and increases with decreasing coupling angle. The frequency dependence of G was summarized for various reciprocal and nonreciprocal multiport networks.

Appendix

Equations (6) and (8)–(10) require the eigenimpedances of a symmetrical six-port network. Davies and Cohen have given expressions for these eigenimpedances [3] namely

$$\frac{Z_i(m)}{Z_0}=-j\cot\left(\frac{\psi_i}{2}\right)$$

$$=\frac{-jZ_{\text{eff}}}{Z_d}\frac{m\psi}{\pi}\sum_{n=mp+i}\left(\frac{\sin n\psi}{n\psi}\right)^2\cdot\frac{J_n(kR)}{\left[(\kappa/\mu)(n/kR)J_n(kR)-J_n'(kR)\right]} \tag{14}$$

for $p=0,\pm1,\pm2,\cdots$, where $m=6$ for a six port, and $i=0$, ±1, ±2, and 3 give the eigenimpedances. Note that

$$Z_0(3)=\frac{Z_0(6)+Z_3(6)}{2}$$

$$Z_{-1}(3)=\frac{Z_{-1}(6)+Z_2(6)}{2}$$

$$Z_1(3)=\frac{Z_1(6)+Z_{-2}(6)}{2}.$$

$Z_i(6)$ can be obtained by omitting the appropriate terms in the expansions for $Z_i(3)$. Using the identities

$$J_n'(kR)=\frac{nJ_n(kR)}{kR}-J_{n+1}(kR)$$

$$J_{-n}(kR)=(-1)^nJ_n(kR)$$

(14) reduces to

$$\frac{Z_i(m)}{Z_0}=-j\frac{Z_{\text{eff}}}{Z_d}\frac{m\psi}{\pi}\cdot\sum_{n=mp+i}\left(\frac{\sin n\psi}{n\psi}\right)^2\frac{1}{\kappa/\mu\dfrac{n}{kR}-\dfrac{|n|}{kR}+\dfrac{J_{|n+1|}(kR)}{J_{|n|}(kR)}}. \tag{15}$$

For a given value of kR,

$$\lim_{n\to\infty}\frac{J_{|n+1|}(kR)}{J_{|n|}(kR)}=0.$$

It is apparent that this series must converge if the series $\sum_{n=1}^{\infty}1/n^3$ converges, which it does.

References

[1] H. Bosma, "On stripline Y-circulation at UHF," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-12, pp. 61–72, Jan. 1964.
[2] G. P. Riblet, "The measurement of the equivalent admittance of 3-port circulators via an automated measurement system," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 401–405, May 1977.
[3] J. B. Davies and P. Cohen, "Theoretical design of symmetrical junction stripline circulators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-11, pp. 506–512, Nov. 1963.
[4] G. P. Riblet, "The extent of the similarity between below resonance and above resonance operation of standard circulator junctions," in *Proc. 1978 IEEE MTT-S Int. Microwave Symp.*, pp. 323–325, June 1978.
[5] Y. S. Wu and F. J. Rosenbaum, "Wide band operation of microstrip circulators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 849–856, Oct. 1974.
[6] G. P. Riblet, "A coupling theorem for matched symmetrical two branch four-port networks," *IEEE Trans. Circuits Syst.*, vol. CAS-25, pp. 145–148, Mar. 1978.
[7] J. Helszajn, *Passive and Active Microwave Circuits.*, New York: Wiley, 1978.